Glossary of Notation

$\lvert \cdot \rvert$	absolute value, or cardinality (of a set), or determinant (of a matrix)
$\aleph_0, \aleph_1, \ldots$	different cardinalities of infinity, $\aleph_0 = \lvert \mathbb{N} \rvert$, $\aleph_1 = \lvert \mathcal{P}(\mathbb{N}) \rvert = \lvert \mathbb{R} \rvert = \mathfrak{c}$
$\alpha(G)$	independence number of graph G, the size of a maximum independent set
$O(\cdot)$	big-oh, asymptotically smaller than a constant multiple of argument
$\Omega(\cdot)$	big-omega, asymptotically larger than a constant multiple of argument
$\Theta(\cdot)$	big-theta, asymptotically comparable to a constant multiple of argument
$o(\cdot)$	little-oh, asymptotically negligible compared to argument
$\omega(\cdot)$	little-omega, asymptotically infinitely larger than argument
$*$	beacon symbol on a TM's tape
Bernoulli(p)	Bernoulli distribution, 1 with probability p and 0 with probability $1-p$
$\mathcal{B}$	all finite binary strings, including the empty string ε, $\mathcal{B} = \{\varepsilon, 0, 1, 00, 01, 10, 11, 000, 001, \ldots\}$
$\mathcal{B}_\infty^0$	all infinite binary strings that are eventually 0 (after some point all bits are 0), e.g. $010000\cdots$
$\mathcal{B}_\infty$	all infinite binary strings
$\tilde{\mathcal{B}}_\infty$	all infinite binary strings that are not eventually 0, there is no point after which all bits are 0
$B(k; n, p)$	Binomial probability of k heads in n coin flips with $\mathbb{P}[\mathrm{H}] = p$, $B(k; n, p) = \binom{n}{k} p^k (1-p)^k$
$B(n, p)$	binomial distribution with n flips and success probability p, $\mathbb{P}[\mathbf{X} = k] = \binom{n}{k} p^k (1-p)^{n-k}$
$\binom{r}{k}$	generalized Binomial coefficient for arbitrary r, $\binom{r}{k} = r^{\underline{k}}/k! = r(r-1)(r-2)\cdots(r-k+1)/k!$
$[\![\cdot]\!]$	Boolean indicator function equal to 1 if the argument is true, and 0 if false
$A \times B$	cartesian product, 2-tuples (a, b) where $a \in A$ and $b \in B$(e.g. $\mathbb{N} \times \mathbb{Z}$)
X^d	d-tuples $(x_1, \ldots, x_d)$ where $x_i \in X$, e.g. $\mathbb{R}^d$
C_n	nth Catalan number, $C_n = \frac{1}{n+1}\binom{2n}{n}$
CDF	cumulative distribution function, probability to be at most a specific value
CFG	context free grammar
CFL	context free language, a language generated by a CFG
$\chi, \chi(G)$	chromatic number of graph G, the minimum number of colors that produce a valid coloring
$\binom{n}{k}$	Binomial coefficient n-choose-k (number of k-combinations from n items), $\binom{n}{k} = n!/k!(n-k)!$
$\binom{n}{k_1,\ldots,k_r}$	multinomial coefficient, $\binom{n}{k_1,\ldots,k_r} = n!/k_1!k_2!\cdots k_r!$
CLT	Central Limit Theorem
$w_1 \bullet w_2$	concatenation, e.g. $01 \bullet 10 = 0110$
$w^{\bullet k}$	concatenated power, e.g. $(01)^{\bullet 2} = 0101$
coNP	problems whose NO-answer is polynomialy verifiable
$\mathfrak{c}$	cardinality of the continuum, $\mathfrak{c} = \lvert \mathbb{R} \rvert$
C_n	cycle graph on n vertices
d-PDA	deterministic pushdown automaton, a DFA with stack memory
δ_i	degree of vertex v_i in a graph with vertices $\{v_1, \ldots, v_n\}$
$\Delta, \Delta(G)$	maximum degree in a graph G, $\Delta = \max_i \delta_i$
$\boldsymbol{\delta}$	degree sequence $\boldsymbol{\delta} = [\delta_1, \ldots, \delta_n]$, the degrees of all the vertices $\{v_1, \ldots, v_n\}$
D_n	number of derangements of n items, $D_n = n! \sum_{k=0}^{n} (-1)^k/k!$
$D_{n,k}$	number of partial derangements of n items, with k staying in place
$\frac{d(\cdot)}{dx}$	derivative with respect to x

 GLOSSARY OF NOTATION

DFA	deterministic finite automaton
$d(\cdot,\cdot)$	distance between two objects, e.g. $d(v_1, v_2)$ is the distance between two vertices in a graph
$d\|n$	d divides n, which means $n = kd$ for $k \in \mathbb{Z}$
e	universal constant, Euler's number (base of natural logarithms), $e = 2.718281828\cdots$
$\varnothing, \{\}$	empty set, containing no elements
E	edge set of graph, for example $E = \{(v_1, v_2), (v_3, v_6)\}$
ε	empty string, containing non symbols, similar to the empty set $\varnothing$
$\langle\cdot\rangle$	encoding of some object, usually into binary, e.g. $\langle M\rangle$ encodes the TM M to a binary string
$\textsc{e}$	evidence used by a certifier for an NP-prooblem
$\exists$	existential quantifier THERE EXISTS, e.g. $\exists x : 3x > x^2$
e^x	exponent of x in the base $e = 2.71828\cdots$
$\mathbb{E}[\mathbf{X}\|A]$	conditional expectation of $\mathbf{X}$ given event A
$\mathbb{E}[\mathbf{Y}\|\mathbf{X}]$	conditional expectation of $\mathbf{Y}$ given $\mathbf{X}$
$\mathbb{E}[\cdot]$	expected value of argument
$\mathbb{E}_{\mathbf{X}}[\cdot]$	expected value with respect to the random variable $\mathbf{X}$
$F_{\mathbf{X}}(x)$	cumulative probability distribution of $\mathbf{X}$, $F_{\mathbf{X}}(x) = \mathbb{P}[\mathbf{X} \le x]$
$k!$	factorial, $k! = k(k-1)(k-2)\cdots 3 \times 2 \times 1$
$x^{\underline{k}}$	factorial or falling power, $x^{\underline{k}} = x(x-1)(x-2)\cdots(x-k+1)$, e.g. $6^{\underline{2}} = 30$
F_n	nth Fibonacci number, $F_1 = F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n > 2$
$\forall$	universal quantifier FOR ALL, e.g. $\forall x : x^2 > 0$
$g \circ f(x)$	composition of g with f, $g \circ f(x) = g(f(x))$
$f \otimes g(x)$	cartesian product function, $f \otimes g(x) = (f(x), g(x))$
γ	universal Euler-Macheroni constant, the $\gamma = \lim_{n\to\infty} H_n - \ln n = 0.577215664\cdots$
$\gcd(m,n)$	greatest common divisor of m and n, e.g. $\gcd(27,12) = 3$
$G(s)$	generating function for sequence $A_0, A_1, A_2, A_3, \ldots$, $G(s) = A_0 + A_1 s^1 + A_2 s^2 + A_3 s^3 + \cdots$
$G = (V,E)$	graph G with vertices V and edges E; usually $n = \|V\|$ and $m = \|E\|$
$\overline{G}$	complement graph, where the edge set of $\overline{G}$ is the complement of the edge set E of G
H_k	Hadamard matrix, a $2^k \times 2^k$ orthogonal matrix whose entries are all $\pm 2^{-k/2}$
H_n	nth harmonic number, $H_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n}$
$\textsc{h}(w)$	hash function, $\textsc{h}(w)$ maps w uniformly to a rndom number in a range $[0, L]$
$\mathrm{height}(T)$	height of rooted tree T, length of the longest path from the root to a leaf
$\text{in-deg}(v_i)$	in-degree of vertex v_i in a directed graph
$\int$	integral
$[a,b]$	interval of real numbers from a to b, where the context indicates $a \le b$ are real
$[i,j]$	interval of integers when $i \le j$ are integers, e.g. $[4,7] = \{4,5,6,7\}$
K_n	clique or complete graph on n vertices
$K_{n,m}$	complete bipartite graph on a bipartite graph with partitions of sizes n and m
$\mathcal{K}(x)$	Kolmogorov complexity, shortest description of string x
$\mathcal{L}$	language, set of finite strings over an alphabet Σ
$\mathcal{L}_1 \bullet \mathcal{L}_2$	concatenation of languages, strings formed by appending a string in $\mathcal{L}_2$ to a string in $\mathcal{L}_1$
$\mathcal{L}^{\bullet k}$	strings formed by concatenating k strings from $\mathcal{L}$
$\mathcal{L}^*$	Kleene-star, all possible concatenations of strings from $\mathcal{L}$, including the empty string ε
$\mathcal{L}^+$	all positive or non-empty concatenations of strings from $\mathcal{L}$ (excludes the empty string ε)
$\mathcal{L}_{\textsc{halt}}$	language of halting TMs $\{\langle M\rangle\#w\}$, where M is a TM that halts on w
$\mathcal{L}_{\textsc{tm}}$	language of successful TMs $\{\langle M\rangle\#w\}$, where M is a TM that accepts w
$\mathcal{L}^{\textsc{r}}$	language reversal, reversal of the strings in $\mathcal{L}$
$\mathrm{lcm}(m,n)$	least common multiple of m and n, e.g. $\gcd(2,3) = 6$
LHS	left hand side, for example in an equation, inequality or implication
$\mathrm{length}(w)$	length of the string w (number of symbols), e.g. $\mathrm{length}(1101) = 4$
$L(G)$	line graph of G (edges become vertices, and edges incident with the same vertex are linked)
$\ln$	logarithm in base $e = 2.71828\cdots$
L_n	path or line on n vertices

$\log_{10}$	logarithm in base 10
$\log_2$	logarithm in base 2
$\wedge$	logical AND, e.g. $p \wedge q \wedge r$
$\stackrel{\text{eqv}}{\equiv}$	logical equivalence, e.g. $p \stackrel{\text{eqv}}{\equiv} q$ means p is equivalent to q
$\leftrightarrow$	logical if and only if (equivalence), e.g. $p \leftrightarrow q$ means p IF AND ONLY IF q
$\rightarrow$	logical implication, e.g. $p \rightarrow q$ means p IMPLIES q, or IF p, THEN q
$\vee$	logical OR, e.g. $p \vee q \vee r$
$a \oplus b$	bitwise OR, e.g. $0001110010 \oplus 1000111000 = 1001111010$
μ	mean (expected value) of a random variable
$A \stackrel{\text{BIJ}}{\mapsto} B$	mapping A bijectively to B, i.e. 1-to-1 and onto, so $\lvert A\rvert = \lvert B\rvert$
$A \stackrel{\text{INJ}}{\mapsto} B$	mapping A injectively to B, i.e. 1-to-1, so $\lvert A\rvert \leq \lvert B\rvert$
1-to-1	mapping that distinct inputs to distinct outputs, an injection
onto	mapping that uses all possible outputs (every output is mapped to by some input), a surjection
$A \stackrel{\text{SUR}}{\mapsto} B$	mapping A surjectively to B, i.e. onto, so $\lvert A\rvert \geq \lvert B\rvert$
$\max(\cdot,\cdot)$	maximum of the two arguments, e.g. $\max(1,2) = 2$
$\min(\cdot,\cdot)$	minimum of the two arguments, e.g. $\min(1,2) = 1$
$a \equiv b \pmod d$	a is congruent to b modulo d, or $d \mid (a-b)$, e.g. $2 \equiv 12 \pmod 5$
$[n]$	in the context of counting, the set $\{1, 2, \ldots, n\}$, e.g. $[4] = \{1, 2, 3, 4\}$.
$\neg(\cdot)$	negation, for example $\neg p$ means NOT(p)
$N(v)$	neighborhood of vertex v, the vertices to which v has an edge
NO	rejecting output of a decider when testing a string for membership in a language
$\lVert \cdot \rVert$	Euclidean norm, $\lVert \mathbf{x} \rVert = (x_1^2 + \cdots + x_d^2)^{1/2}$
NP	nondeterministic polynomial, problems whose YES-answer is polynomialy verifiable on a TM
$\nu_p(x)$	largest power of prime p that divides x, $x = \prod_{\text{primes } p} p^{\nu_p(x)}$, e.g. $\nu_3(18) = 2$
$\mathbb{N}$	natural numbers, $1, 2, 3, \ldots$
$\mathbb{Q}$	rationals, ratio of an integer over a natural number
$\mathbb{R}$	real numbers
$\mathbb{Z}$	integers, $0, \pm 1, \pm 2, \pm 3, \ldots$
ω	outcome of a random experiment
Ω	sample space (possible outcomes)
$<, \leq, >, \geq$	binary ordering relations
out-deg(v_i)	out-degree of vertex v_i in a directed graph
P	polynomialy solvable problems on a TM
PDF	probability distribution function, probability to be a specific value
φ	golden ratio, $\varphi = \frac{1}{2}(1+\sqrt{5}) = 1.618\cdots$
$\phi(x)$	normal CDF, $\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} ds\ e^{-\frac{1}{2}s^2}$
$\phi(n)$	Euler's totient function, the number of co-prime positive divisors of n, e.g. $\phi(9) = 6$
π	universal constant ratio of circumference to diameter, $\pi = 3.141592654\cdots$
$\mathcal{P}(A)$	power set of A containing all subsets of A including the empty set $\varnothing$
$P(n)$	predicate stating a claim depending on n, often used to define an induction claim
$\mathbb{P}[A]$	probability of event A
$P(\omega)$	probability function defined for outcomes in the outcome space, $\omega \in \Omega$
$\mathbb{P}[A \cap B]$	probability of event A and event B occuring
$\mathbb{P}[A \mid B]$	probability of event A conditioned on event B occuring, $\mathbb{P}[A \mid B] = \mathbb{P}[A \cap B] / \mathbb{P}[B]$
$P_{\mathbf{X}}(x)$	probability distribution of $\mathbf{X}$, $P_{\mathbf{X}}(x) = \mathbb{P}[\mathbf{X} = x]$
$P_{\mathbf{XY}}(x,y)$	joint probability distribution of $\mathbf{X}$ and $\mathbf{Y}$
$\prod$	product, e.g. $\prod_{i=1}^{10} i^2 = 1 \times 4 \times 9 \times \cdots \times 100$
q_i	state in a DFA or a TM
$R(k,s)$	Ramsey number, the minimum number of vertices to guarantee a k-clique or an s-war
$\mathcal{L}_1 \leq_{\mathrm{P}} \mathcal{L}_2$	$\mathcal{L}_1$ is polynomialy-reducible to $\mathcal{L}_2$, that is $\mathcal{L}_1$ is polynomialy solvable if $\mathcal{L}_2$ is
$\mathcal{L}_1 \leq_{\mathrm{R}} \mathcal{L}_2$	$\mathcal{L}_1$ is TM-reducible to $\mathcal{L}_2$, that is $\mathcal{L}_1$ is solvable if $\mathcal{L}_2$ is

$\text{rem}(n,d)$	remainder when n is divided by d where $0 \leq \text{rem}(n,d) < d$, e.g. $\text{rem}(22,6) = 4$
w^{R}	reversal of string w, e.g. $001^{\text{R}} = 100$
RHS	right hand side, for example in an equation, inequality or implication
RBT	rooted binary tree, each vertex has at most two children
RFBT	rooted full binary tree, each vertex is a leaf or has two children
RFTT	rooted full ternary tree
RST	rooted short tree
RT	rooted tree
RTT	rooted ternary tree, each vertex has at most three children
$\{x\}$	round to nearest integer ($\frac{1}{2}$ is rounded up), e.g. $\{1.6\} = 2$, $\{1.1\} = 1$ and $\{-1.1\} = -1$
$\lceil x \rceil$	round up, i.e. ceiling, e.g. $\lceil 1.1 \rceil = 2$ and $\lceil -1.1 \rceil = -1$
$\lfloor x \rfloor$	round down, i.e. floor, e.g. $\lfloor 1.1 \rfloor = 1$ and $\lfloor -1.1 \rfloor = -2$
$\{\cdots\}$	set of elements enclosed within curly parentheses, e.g. $\{1,2,3\}$
$\overline{(\cdot)}$	complement of a set, e.g. $\overline{A} = \{\text{items not in } A\}$, or negation of a variable, e.g. $\overline{x} = \text{NOT}(x)$
$\overline{b}$	flipped binary variable b, with 1's replaced by 0's and *vice versa*, e.g. $\overline{0100} = 1011$
$A \cap B$	A intersected with B, the elements in both A and B, e.g. $\{1,2\} \cup \{2,3\} = \{2\}$
$A \setminus B$	A with the elements of B removed, e.g. $\{1,2\} \setminus \{2,3\} = \{1\}$
$A \cup B$	A union B, the elements of A and B combined, e.g. $\{1,2\} \cup \{2,3\} = \{1,2,3\}$
$\sigma^2, \sigma^2(\mathbf{X})$	variance of a random variable $\mathbf{X}$
Σ	alphabet (also used for summation), e.g. $\Sigma = \{0,1\}$ is the binary alphabet
Σ^+	all nonempty strings that can be formed using an alphabet Σ
Σ^*	all strings that can be formed using an alphabet Σ, e.g. for $\Sigma = \{0,1\}$, Σ^* is all binary strings
$\text{size}(T)$	size of rooted tree T, equal to the number of vertices
S_n	star graph on n vertices; also used for the sum up of a series up to n terms
$\textcircled{A}$, $\textcircled{E}$	halting state of a TM for accept and reject (error)
$\rightarrow\bigcirc$	start state in a DFA or TM
$\circledcirc$	yes-state in a DFA, states without the green border are no-states
$\left[{n \atop k}\right]$	Stirling number of the first kind, partitions of n items into k non-empty labeled sets
$\left\{{n \atop k}\right\}$	Stirling number of the second kind, partitions of n items into k non-empty unlabeled sets
$\sum_i^j$	summation, e.g. $\sum_{i=1}^{10} i^2 = 1 + 4 + 9 + \cdots + 10^2$
$\sigma(n)$	sum of the divisors of n, including n, e.g. $\sigma(6) = 12, \sigma(12) = 28$
$\checkmark_0, \checkmark_1$	marked symbols on a TM's tape
#	punctuation symbol used to separate different parts of strings
$\sqcup$	blank symbol, one of the default symbols on a TM's tape
$T(n)$	runtime of an algorithm or Turing Machine on an input of size n
TM	Turing machine
U_{TM}	universal TM
$U[n]$	uniform distribution on $1,\ldots,n$, $\mathbb{P}[\mathbf{X} = k] = 1/n$ for $k = 1,\ldots,n$
$\text{value}(b)$	integer value of binary string $b = b_k b_{k-1} \cdots b_1 b_0$, e.g. $\text{value}(101) = 5$
$\text{var}(\mathbf{X})$	variance of a random variable $\mathbf{X}$
V	vertex set of graph
$W(k,\ell)$	Expected wait to k successes and ℓ failures in n independent trials with success probability p
W_n	wheel graph on n vertices
$w*$	wildcard symbol, e.g. $1* = 1\bullet\Sigma^*$ is the set of all strings starting with 1
$\mathbf{X}$	random variable, a measurement in an experiment, mathematically a function $\mathbf{X} : \Omega \mapsto \mathbb{R}$
$\mathbf{Y}$	random variable, a measurement in an experiment, mathematically a function $\mathbf{Y} : \Omega \mapsto \mathbb{R}$
(YES)	accepting output of a decider when testing a string for membership in a language
(YES)-set	set of strings for which the answer to a problem is yes, i.e. the strings in the language
$\mathbf{Z}$	random variable, a measurement in an experiment, mathematically a function $\mathbf{Z} : \Omega \mapsto \mathbb{R}$
z-score	z-score is a random variable $\mathbf{Z}$ derived from $\mathbf{X}$, $\mathbf{Z} = (\mathbf{X} - \mu(\mathbf{X}))/\sigma(\mathbf{X})$
ZFC	Zermelo-Fraenkel set theory with the axiom of choice